Comments on "Dark matter: A phenomenological existence proof"

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Abstract

A recent paper by Ahluwalia-Khalilova [1] is examined where he claims that the standard FRW cosmology with a Λ predicts existence of dark matter without invoking the data on galactic rotation curves and gravitational lensing. However, we find that his claims are not correct. He has already assumed (without realizing) in the very outset what he wants to prove.

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Recently Ahluwalia-Khalilova has claimed [1] that the flat Friedmann-Robertson-Walker (FRW) cosmology with a cosmological constant Λ predicts the existence of dark matter $\Omega_{dm0} \approx 0.22 \pm 0.08$, without invoking the data on galactic rotation curves or gravitational lensing (where, as usual, Ω_i is the energy density of the constituent ρ_i in units of the critical density $3H^2/8\pi G$ and the subscript '0' denotes the value of the quantity at the present epoch). However, this is not a prediction, but just a reflection of our assumption once we consider the existence of a Λ ($\Omega_{\Lambda 0} \approx 0.73$) in a flat universe together with the insufficient visible matter $\Omega_{baryon} \approx 0.05$ to close the universe. However, Ahluwalia-Khalilova also claims that he has not used any input from Λ (except for working in units of $1/\sqrt{\Lambda}$ for convenience). We find that this is not correct. The constraint on Ω_{m0} found by him has not been derived without using $\Omega_{\Lambda 0} \approx 0.73$, as will be explained in the following.

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In the standard FRW cosmology, the present universe is described by the non-relativistic matter (of energy density $\rho_{\rm m}$) and a constant Λ. The dynamics of the evolution is given by the the Einstein field equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_{\rm m} + \frac{\Lambda}{3} - \frac{k}{a^2}, \tag{1}$$

$$\dot{\rho}_{\rm m} + 3\rho_{\rm m}\left(\frac{\dot{a}}{a}\right) = 0, \tag{2}$$

which can be integrated easily to give the age of the universe [2]:

$$t_0 = \frac{1}{H_0}\int_0^\infty \frac{dz}{\sqrt{[\Omega_{\rm m0}(1+z)^5 - (\Omega_{\rm m0} + \Omega_{\Lambda 0} - 1)(1+z)^4 + \Omega_{\Lambda 0}(1+z)^2]}}. \tag{3}$$

It should be noted that at its face value the age t_0 appearing in equation (3) is, de facto, the dynamical age of the universe (the time it took the universe to evolve from the big bang to the present epoch) which is clearly a function of the parameters H_0, $\Omega_{\rm m0}$ and $\Omega_{\Lambda 0}$. As given by equation (3), it *does not* represent the age of the constituents of the universe, like globular clusters or stars. The age of the universe must be more than the age of its constituents (the structures did not start forming at the big bang). Of course one can always check if the age of the universe is consistent with the age of its constituents. However, the age of even the oldest objects in the universe (for example, the globular clusters) should provide only the lower limit.

Further, a given set of the parameters H_0, $\Omega_{\rm m0}$ and $\Omega_{\Lambda 0}$, gives a unique value of t_0 from equation (3), but the reverse case is very degenerate in solutions, even in the above mentioned limiting case. As there are 4 unknowns in equation (3), equating t_0 to the age of a particular object can yield many sets of H_0, $\Omega_{\rm m0}$, $\Omega_{\Lambda 0}$ as solutions. For example, the values $\Omega_{\rm m0} = 0.271 \pm 0.028 = 1 - \Omega_{\Lambda 0}$ (inferred by Astier et al. [3] which is considered by Ahluwalia-Khalilova in his paper) and $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1} (from the HST key project [4]), give a $t_0 = 13.48 \pm 1.88$ Gyr which is consistent with the age of the globular clusters $t_{\rm GC} = 13.5 \pm 1.5$ Gyr used by Ahluwalia-Khalilova. However, another model $\Omega_{\rm m0} = \Omega_{\Lambda 0} = 0$ with the same value of H_0 also gives a $t_0 = 13.58 \pm 1.51$ Gyr which is consistent with the value of $t_{\rm GC}$ mentioned above. Further, one can find many other solutions permitted by the uncertainties in the observed values of the parameters.

Even after restricting ourselves to the flat models $\Omega_{\mathrm{m}} + \Omega_{\Lambda} = 1$ (as has been done by Ahluwalia-Khalilova), the solutions will be degenerate in 2 parameters. Thus, just by setting a value to t_0, one cannot solve for $\Omega_{\mathrm{m}0}$ (or $\Omega_{\Lambda 0}$) uniquely unless one also sets some value to H_0 (possibly from some other observation). Thus all one can do is one can check whether a particular value of t_0 (say, the age of the globular clusters) is consistent with a model with a certain $\Omega_{\mathrm{m}0}$ together with a certain H_0. However, this cannot be regarded as a prediction or a proof of existence of that $\Omega_{\mathrm{m}0}$. It is just a consistency check, as mentioned earlier. Ahluwalia-Khalilova has also done a similar consistency check, as will be explained in the following, though he does not realize it.

Ahluwalia-Khalilova has considered, in his paper [1], the analytical solution of equations (1) and (2) available in the literature, viz.

$$a^3(t) = \frac{8\pi G \rho_{\mathrm{m}0} a_0^3}{\Lambda} \sinh^2\left(\frac{\sqrt{3\Lambda}}{2}t\right), \tag{4}$$

for the case $k = 0$. By the use of the solution of the conservation equation (2) (i.e., $\rho_{\mathrm{m}} a^3 = \text{constant} = \rho_{\mathrm{m}0} a_0^3$, say), equation (4) can be put in the form

$$t = \frac{2}{\sqrt{3\Lambda}} \sinh^{-1} \sqrt{\frac{\Omega_{\Lambda}(t)}{\Omega_{\mathrm{m}}(t)}}, \tag{5}$$

giving

$$t_0 = \frac{2}{\sqrt{3\Lambda}} \sinh^{-1} \sqrt{\frac{1}{\Omega_{\mathrm{m}0}} - 1}. \tag{6}$$

Now Ahluwalia-Khalilova claims that in order to evaluate $\Omega_{\mathrm{m}0}$ from equation (6), he uses the only input from the observations as $t_0 = t_{\mathrm{GC}}$, and does not take any input from Λ except for working in units of $1/\sqrt{\Lambda}$ (≈ 9 Gyr) for convenience. However this is not correct. He does use the observed value of Λ in equation (6) without realizing it, as we shall see shortly.

If one wants to work in units of $1/\sqrt{\Lambda}$ for the sake of convenience, the proper way is to consider $1/\sqrt{\Lambda}$ in units of 10^9 years. So, let us consider $1/\sqrt{\Lambda} = \lambda \times 10^9$ years, where λ is the dimensionless value of $1/\sqrt{\Lambda}$ in units of 10^9 years. Equation (6) then gives

$$\Omega_{\mathrm{m}0} = \left[1 + \sinh^2\left\{\frac{\sqrt{3}}{2\lambda}(13.5 \pm 1.5)\right\}\right]^{-1}. \tag{7}$$

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Now, can one evaluates the r.h.s. of equation (7) without putting a numerical value of λ and can get $\Omega_{m0} = 0.27 \pm 0.08$, as obtained by Ahluwalia-Khalilova? The answer is a flat no. So, a value of Λ has been assumed by him a priori!

It should be worthwhile here to comment on the observational status of Λ. It should be noted that Λ (or any other variant of dark energy) is not observed directly like the visible matter density (ρ_{baryon}). Rather it is inferred from the observations by comparing the distance (luminosity distance or angular diameter distance) of an object predicted by a cosmological model to the observed distance given by the observations of supernovae of type Ia, cosmic microwave background or gravitational lensing. One can also infer bounds on it by comparing the dynamical age of the universe to the age of the oldest objects in it. These methods estimate $\Omega_{\Lambda 0}$, from which Λ is deduced by using the observed value of the Hubble parameter H_0. Thus, in quoting a value for Λ from the observations, there is always an associated H_0 tacitly assumed. This fact is reflected when one writes equation (6) in the following alternative form by using the relations $\Lambda/3H_0^2 = \Omega_{\Lambda 0} = 1 - \Omega_{m0}$:

$$t_0 = \frac{2}{3H_0\sqrt{1 - \Omega_{m0}}} \sinh^{-1}\sqrt{\frac{1}{\Omega_{m0}} - 1}, \qquad (8)$$

which gives the same answer as equation (3) for the case $k = 0$.

Hence it is clear that by assuming a Λ (from $1/\sqrt{\Lambda} \approx 9$ Gyr), Ahluwalia-Khalilova has already assumed the existence of an $\Omega_{\Lambda 0} \approx 0.73$ (for $H_0 \approx 72$ km s^{-1} Mpc^{-1}) and hence an $\Omega_{m0} = 1 - \Omega_{\Lambda 0}$ (≈ 0.27). It is already known that these values give the correct age of the universe. Thus the exercise done by him is simply a consistency check of the standard cosmology and does not provide any existence proof for Ω_{m0} and hence for dark matter (in view of the insufficient $\Omega_{baryon} \approx 0.05$ to close the universe).

It should also be noted that the value $\Lambda/(8\pi G) \approx 4 \times 10^{-47}$ GeV^4 does not seem to give $\tau_\Lambda \equiv 1/\sqrt{\Lambda} \approx 9$ Gyr, as used by Ahluwalia-Khalilova. Rather it should be replaced by $\Lambda/(8\pi G) \equiv \rho_v \approx 3 \times 10^{-47}$ GeV^4 (in units of $c = 1$, $\hbar = 1$).

References

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